

1. On the vector space X , suppose there are two norms $\|\cdot\|_1$ and $\|\cdot\|_2$ on it. If these two norms are equivalent, prove that $(X, \|\cdot\|_1)$ is complete if and only if $(X, \|\cdot\|_2)$ is complete.

2. On finite dimensional vector space \mathbb{R}^n , define the norm as

$$\|x\| = |x_1| + \cdots + |x_n| .$$

a) Prove that the above defined norm is well-defined. In other words, it satisfies the definition of norm (the properties of norm).

b) Prove that under the above defined norm, $(\mathbb{R}^n, \|\cdot\|)$ is complete. (You can use the following fact without needing to prove it: \mathbb{R} is complete)